2.3 Evaluation of limits to infinity

2014-09-15 day 14

2.3/19

$$\lim_{x\to-\infty} \frac{\sqrt{5x^2-2}}{x+3}$$ indeterminate of type $\frac{\infty}{\infty}$

$$= \lim_{x\to-\infty} \frac{\sqrt{x^2}\sqrt{5-\frac{2}{x^2}}}{x\left(1+\frac{3}{x}\right)} = \lim_{x\to-\infty} \frac{-x\sqrt{5-\frac{2}{x^2}}}{x\left(1+\frac{3}{x}\right)}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{5-\frac{2}{x^2}}}{1+\frac{3}{x}} = \frac{-\sqrt{5}}{1}$$

$$\sqrt{x^2} = |x|$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

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2014-09-15 day 14

is x positive or negative?

Idk.

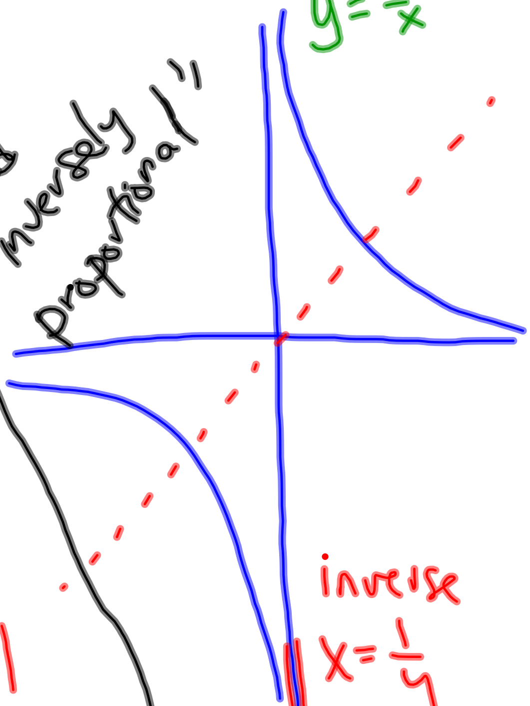
$\frac{1}{-10} = -.1$

$\frac{1}{-100} = -.01$ called "inversely proportional"

$\frac{1}{-10^3} = -.001$

$\frac{1}{-10^4} = -.0001$

$\vdots \qquad \vdots$



inverse

$x = \frac{1}{y}$

$xy = 1$

$y = \frac{1}{x}$

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2014-09-15 day 14

15) $$\lim_{x \to -\infty} \frac{x-2}{x^2+2x+1} = \text{indeterminate } \frac{\infty}{\infty}$$

$$\lim_{x \to -\infty} \frac{x\left(1-\frac{2}{x}\right)}{x\left(x+2+\frac{1}{x}\right)} = \lim_{x \to -\infty} \frac{1-\frac{2}{x}}{x+2+\frac{1}{x}}$$

$$\approx \frac{1 \text{ ish}}{-\infty \text{ ish}} = 0$$

$$\begin{array}{r} x+4 \\ x-2 \overline{\big) x^2+2x+1} \\ -(x^2-2x) \\ \hline 4x+1 \\ -(4x-8) \\ \hline 9 \end{array}$$

$$\lim_{x \to -\infty} \frac{\cancel{(x-2)}^{\,1}}{\cancel{(x-2)}\left(x+4+\frac{9}{x-2}\right)} = 0$$

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2014-09-15 day 14

1) $\lim_{x\to+\infty} \sqrt[3]{\dfrac{2+3x-5x^2}{1+8x^2}} = \lim_{x\to\infty} \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \dfrac{\sqrt[3]{\frac{2}{x^2}+\frac{3}{x}-5}}{\sqrt[3]{\frac{1}{x^2}+8}}$

$$☆ = \sqrt[3]{\lim_{x\to+\infty} \frac{x^2\left(\frac{2+3x-5x^2}{x^2}\right)}{x^2\left(\frac{1+8x^2}{x^2}\right)}} = \sqrt[3]{-\frac{5}{8}} = \frac{-\sqrt[3]{5}}{2}$$

The end behavior of a polynomial is the same as the end behavior of the high degree term.

$$☆ = \sqrt[3]{\lim_{x\to+\infty} \frac{-5x^2}{8x^2}} = \cdots$$

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2014-09-15 day 14

26) $$\lim_{x\to-\infty} \frac{5-2x^3}{x^2+1}$$

predict:
$$\lim_{\sim} \sim = +\infty$$

$$= \lim_{x\to-\infty} \frac{x^3\left(\frac{5}{x^3}-2\right)}{x^3\left(\frac{1}{x}+\frac{1}{x^3}\right)} = \lim_{x\to-\infty} \frac{\frac{5}{x^3}-2}{\frac{1}{x}+\frac{1}{x^3}}$$

$$\approx \frac{-2\text{ ish}}{0\text{ ish}} \left\{ \begin{array}{l} +\infty \\ -\infty \\ \text{DNE} \end{array} \right.$$

$$\lim_{x\to-\infty} \frac{5-2x^3}{x^2+1}$$

$$= \lim_{x\to-\infty} \frac{x^2\left(\frac{5}{x^2}-2x\right)}{x^2\left(1+\frac{1}{x^2}\right)} = \lim_{x\to-\infty} \frac{\frac{5}{x^2}-2x}{1+\frac{1}{x^2}}$$

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2014-09-15 day 14

31) $\lim_{x \to +\infty} \frac{\sqrt{x^2+3} - x}{1} \cdot \frac{\sqrt{x^2+3} + x}{\sqrt{x^2+3} + x}$

indeterminate form of $\infty - \infty$

$= \lim_{x \to \infty} \frac{(x^2+3) - x^2}{\sqrt{x^2+3} + x}$

$= \lim_{x \to \infty} \frac{3}{\sqrt{x^2+3} + x} = 0$